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13. ABSTRACT *(Maximum 200 words)*

A number of distinct treatments of transversely varying thickness modes in contoured quartz resonators with beveled edges have been performed. In the first two of these treatments the conditions at the free-edge of the resonator were ignored. The first of the two employed rectangular coordinates while the second employed the more suitable cylindrical coordinates. In the third treatment the dominant condition at the free-edge of the cylindrical resonator was included. This latter treatment resulted in an analytical-computational procedure for the estimation of the diameter required for a good resonator. In other work the influence of an asymmetric air-

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gap on transversely varying thickness modes in quartz trapped energy resonators was determined and the equation for transversely varying thickness modes was modified to account for the gap. In different work the similarities and differences in the analytical-computational description of the acceleration sensitivities of acoustic bulk and surface wave resonators were noted and compared.

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   2. "The Influence of an Asymmetric Air Gap on the Vibration Characteristics of Quartz Trapped Energy Resonators", H. F. Tiersten, Proceedings of the 1995 IEEE International Frequency Control Symposium, IEEE Catalog Number 95 CH 3575-2, Institute of Electrical and Electronics Engineers, New York, 740-745 (1995).

   3. "On the Accurate Description of Piezoelectric Resonators Subject to Biasing Deformations", H. F. Tiersten, International Journal of Engineering Science, 33, 2239-2259 (1995).

   4. "Transversely Varying Thickness Modes in Trapped Energy Resonators with Shallow and Beveled Contours", H. F. Tiersten, B. J. Lwo and B. Dulmet, Journal of Applied Physics, 80, 1037-1046 (1996).

   5. "Similarities and Differences in the Analytical Descriptions of the Acceleration Sensitivities of Acoustic Bulk and Surface Wave Resonators", H. F. Tiersten, Proceedings of the 1996 IEEE International Frequency Control Symposium, IEEE Catalog Number 96 CH 35935, Institute of Electrical and Electronics Engineers, New York, 430-438 (1996).

   6. "The Influence of the Free Edge on the Vibration Characteristics of a Contoured, Beveled Cylindrical Quartz Resonator", J. S. Yang and H. F. Tiersten, Proceedings of the 1996 IEEE International Frequency Control Symposium, IEEE Catalog Number 96 CH 35935, Institute of Electrical and Electronics Engineers, New York, 657-664 (1996).

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BRIEF OUTLINE OF RESEARCH FINDINGS

An analysis of transversely varying thickness modes in contoured quartz resonators with beveled cylindrical edges has been performed. The radial dependence of the eigensolution in the inner electroded region was obtained by the method of Fröbenius and in the outer unelectroded region by means of an asymptotic expansion about a point just outside the edge of the electrodes. The resonant frequencies of some harmonics and anharmonics of SC- cut quartz resonators including a TCXO were calculated along with the motional capacitance of harmonics and anharmonics which have no angular difference. Comparisons were made with an earlier treatment in which a rectilinear approximation was made. The influences on the mode shapes of both the inner and outer radii of curvature of the contours and the portions over which each was operative were exhibited.

An analysis of the influence of an asymmetric air gap on transversely varying thickness modes in quartz trapped energy resonators was performanced. The thickness-dependent problem that transforms the inhomogeneity from the electrodes into the equation for transversely varying thickness modes was identified and solved. It was shown that the asymmetry of the gap has no influence on either the resonant frequencies or motional and static capacitances of the resonators and only the mean value of the gap does. The equation for transversely varying thickness modes was modified to account for the gap.

The advantage and importance of using rotationally invariant nonlinear electroelastic equations for the description of piezoelectric resonators subject to biasing deformations (even homogeneous thermal) was shown[3]. This is a result of the fact that the general nonlinear description and even its linear limit can be referred to reference coordinates, to which an intrinsically linear description cannot. As a consequence, when the nonlinear description or its linear limit is employed, the influence of biasing states (even homogeneous thermal) can be determined without ever finding the biased geometry. Examples of the additional accuracy obtained with this description were given.

An analysis of transversely varying thickness modes in trapped energy resonators with shallow and beveled contours and rectangular electrodes has been performed. The influence of both the contouring and the continuity conditions at the edge of the electrodes was included in the treatment. It was shown that the treatment should be used for contoured resonators with ordinary contours operating in the fundamental mode when the electrodes are of relatively small size. The results indicate why and when the analysis for the contoured resonator which ignores the continuity conditions at the electrode edges works as well as it does. It is shown that the treatment is applicable to the harmonics of a beveled resonator if the circle of intersection of the inner and outer contours is replaced by the circumscribed square. The influence of the radii of curvature and domain of the inner and outer contours on the mode shape was exhibited.

The similarities and differences in the analytical description of the acceleration sensitivities of acoustic bulk and surface wave resonators have been noted and summarized[5]. The importance and advantages of the use of finite deformation theory over infinitesimal deformation theory for the accurate prediction of the behavior of both precision resonant devices was noted. It was noted that with such a description the influence of biasing states can be determined without ever finding the biased geometry. It was noted that the very useful equation for the accurate calculation of the perturbation in eigenfrequency due to a bias was obtained from the nonlinear description. The detailed analytical descriptions of the mode shapes of both surface wave and contoured resonators were presented and contrasted.

The influence of the free edge on the vibration characteristics of a contoured, beveled cylindrical quartz resonator has been studied[6]. The associated analytical problem, which had been treated in earlier work, had to be reconsidered. The new treatment differs from that of earlier work in that the dominant condition at the edge of the resonator is satisfied and the approximation made earlier in the beveled annulus is not made in the new treatment, in which the solution is exact. An analytical procedure for the determination of the required diameter for a good resonator was identified and calculated results were compared with earlier work.